# Domain size in the presence of random fields

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We consider the size of domains formed in ordered systems in the presence of quenched random fields. We argue that below the critical dimension, the domain size shows a nonmonotonic dependence on the correlation length of the random field. If the random field is slowly varying in space, the order parameter follows the field, and the domain size is comparable to the correlation length. If the field is rapidly varying, the domain size becomes larger than the correlation length, and diverges as the correlation length of the random field goes to zero. [S1063-651X(98)01001-0]

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In a classic paper [1], Imry and Ma showed that even at zero temperature, long range order is destroyed by the presence of arbitrarily weak random fields if the spatial dimension is less than some critical value. They note that the energy cost of domain walls for domains of size $L$ is $\sim L^{d-2}$ for systems with continuous symmetry, such as the Heisenberg model, and $\sim L^{d-1}$ for systems with discrete symmetry, such as the Ising model, where $d$ is the spatial dimension, while the energy gain from the random field is $\sim L^{d/2}$. Thus the formation of domains and the loss of long range order occurs if $d<4$, in systems with continuous symmetry, and $d<2$ in systems with discrete symmetry. In this paper we extend this argument, and consider the dependence of the domain size on the correlation length of the random field. A great deal of work has been done on the influence of quenched disorder on critical behavior [2–6]. Here we concentrate on the ground state configuration.

We assume the free energy density to be of the form

$$F=F_0+\kappa(\nabla\psi)^2-H\psi, \tag{1}$$

where $\psi$ is the dimensionless order parameter (of order unity), $\kappa$ is an elastic constant, and $H$ is a random field with correlation length $\zeta$. The order parameter can be of a general type, and $H\psi$ denotes the inner product in order parameter space. In dimensionless form, the free energy density becomes

$$F=F_0+l_0^2(\nabla\psi)^2-h\psi, \tag{2}$$

where $l_0\sim H^{-1/2}$ is a coherence length, and $h$ is the random field normalized so that $\langle h^2\rangle=1$. For systems with continuous symmetry, the meaning of coherence length $l_0$ is that the field cannot produce deformations of the order parameter on length scales shorter than $l_0$. We show below that the dependence of the domain size $L$ on $\zeta$ is nonmonotonic, and differs in the regimes $\zeta/l_0\gg 1$ and $\zeta/l_0\ll 1$. To determine the ground state, we minimize the total free energy, or equivalently, the average free energy density, with respect to the domain size $L$.

We consider first systems with continuous symmetry, such as the Heisenberg model or nematic liquid crystals. If the order parameter follows the field, the contribution from the field term in Eq. (2) is $\sim -1$, while the average value of the squared gradient term is $\sim(l_0/\zeta)^2$. If $\zeta/l_0\gg 1$, the field term dominates, and hence in this regime the domain size $L$ is comparable to the correlation length $\zeta$. That is, for all spatial dimensions $d$,

$$\frac{L}{l_0}\simeq\frac{\zeta}{l_0} \quad \text{if} \quad \zeta/l_0\gg 1. \tag{3}$$

If $\zeta/l_0\ll 1$, the order parameter cannot follow the field and $L>\zeta$. In this case the average free energy density is

$$\mathcal{F}=\langle F-F_0\rangle\simeq l_0^2L^{-2}-\left(\frac{L}{\zeta}\right)^{d/2}\zeta^dL^{-d}, \tag{4}$$

where $(L/\zeta)^{d/2}$ is the number of correlated regions in a domain of size $L$ where $h\psi\approx 1$. Minimizing $\mathcal{F}$ with respect to $L$ gives

$$\frac{L}{l_0}=\left(\frac{4}{d}\right)^{2/(4-d)}\left(\frac{\zeta}{l_0}\right)^{d/(d-4)} \quad \text{if} \quad \zeta/l_0\ll 1. \tag{5}$$

In this regime, therefore, $L\sim\zeta^{2/(d-4)}$, and the domain size diverges as $\zeta\to 0$. Physically, the correlation length cannot be less than $a/2$ where $a$ is a lattice constant, thus the maximum domain size $L_{\max}\simeq l_0(a/l_0)^{d/(d-4)}\sim H^{2/(d-4)}$ is the Imry-Ma length [6]. The asymptotic behavior is, from Eqs. (3) and (5),

$$\frac{L}{l_0}\simeq\left(\frac{\zeta}{l_0}\right)^{d/(d-4)}+\frac{\zeta}{l_0}. \tag{6}$$

Thus the domain size is comparable to the correlation length of the random field if $\zeta/l_0\gg 1$, but diverges for $d<4$ as $l_0(\zeta/l_0)^{d/(d-4)}$ if $\zeta/l_0\ll 1$. For $d\geq 4$, the free energy is minimized when $L\to\infty$ if $\zeta/l_0\ll 1$.

 

A similar argument follows for the case of discrete symmetry, such as the Ising model. If the order parameter follows the field, the contribution from the field term in Eq. (2) is again $\sim -1$, while the average value of the squared gradient term is $\sim l_0^2/(a\zeta)$, where $a$ is the lattice constant. The lattice constant enters here since the wall thickness is $\sim a$; the coherence length in this case is $l_0' = l_0^2/a$. If $\zeta/l_0' \gg 1$, the field term dominates, and hence in this regime the domain size is expected to be comparable to the correlation length. That is, for all $d$,

$$\frac{L}{l_0'} \simeq \frac{\zeta}{l_0'} \quad \text{if} \quad \zeta/l_0' \gg 1. \tag{7}$$

If $\zeta/l_0' \ll 1$, the order parameter cannot follow the field and $L > \zeta$. In this case the average free energy density is

$$\mathcal{F} \simeq l_0' L^{-1} - \left(\frac{L}{\zeta}\right)^{d/2} \zeta^d L^{-d}. \tag{8}$$

Minimizing $\mathcal{F}$ with respect to $L$ gives

$$\frac{L}{l_0'} = \left(\frac{2}{d}\right)^{2/(2-d)} \left(\frac{\zeta}{l_0'}\right)^{d/(d-2)} \quad \text{if} \quad \zeta/l_0 \ll 1. \tag{9}$$

The asympotic behavior is then, from Eqs. (7) and (9),

$$\frac{L}{l_0'} \simeq \left(\frac{\zeta}{l_0'}\right)^{d/(d-2)} + \frac{\zeta}{l_0'}. \tag{10}$$

Thus the domain size is comparable to the correlation length of the random field if $\zeta/l_0' \gg 1$, but diverges for $d < 2$ as $l_0'(\zeta/l_0')^{d/(d-2)}$ if $\zeta/l_0' \gg 1$. For $d \geq 2$, the free energy is minimized when $L \to \infty$ if $\zeta/l_0' \ll 1$.

The behavior of domain size $L$ as a function of the correlation length $\zeta$ of the random field is shown in Fig. 1 for various values of the coherence length $l_0$ for the continuous symmetry case where $d = 3$. The dependence of the domain size $L$ on random field correlation length $\zeta$ and coherence length $l_0 \sim H^{-1/2}$ is different in the regimes where $\zeta/l_0 \ll 1$ and where $\zeta/l_0 \gg 1$.

As the strength of the field increases, $l_0$ decreases, and the crossover regime moves to shorter length scales.

In summary, we have argued that if the random field is slowly varying in space compared to the coherence length, the domain size is comparable to the correlation length of the random field. When the field is rapidly varying, however, the domain size becomes large, and approaches the Imry-Ma length, which diverges as the strength of the random field goes to zero. This divergence of domain size is consistent with the argument of Imry and Ma [1] and more heuristic arguments presented elsewhere [12]. It is interesting to note that, in the regime $\zeta/l_0 \ll 1$, Eq. (6) gives the Larkin length [13] for the flux lattice in the presence of random pinning centers if the correlation length $\zeta$ is identified as $a^2\rho^{1/d}$, where $\rho$ is the number density of pinning centers. There is no analog of the regime $\zeta/l_0 \gg 1$ in Ref. [13]. By contrast, some experiments on liquid crystals in porous media [9–11] indicate that the domain size $L \sim \zeta$, if the correlation length $\zeta$ is identified as the pore size, as predicted by Eq. (6) in the regime $\zeta/l_0 \gg 1$. Other experiments on liquid crystals [7,8] show domain sizes that are much larger than the pore size. In confined superfluid $^4$He, correlation lengths of the order parameter, both smaller and larger than the pore size, have been measured [14–16]. Although experiments on the above systems are consistent with the predictions of Eq. (6) in different regimes, we are not aware of any one experiment clearly showing the predicted crossover behavior. Monte Carlo simulations with uncorrelated random fields [18,17] indicate order parameter correlation lengths well in excess of the lattice spacing. Numerical simulations to study the dependence of domain size on correlation length of the random field are currently under way.

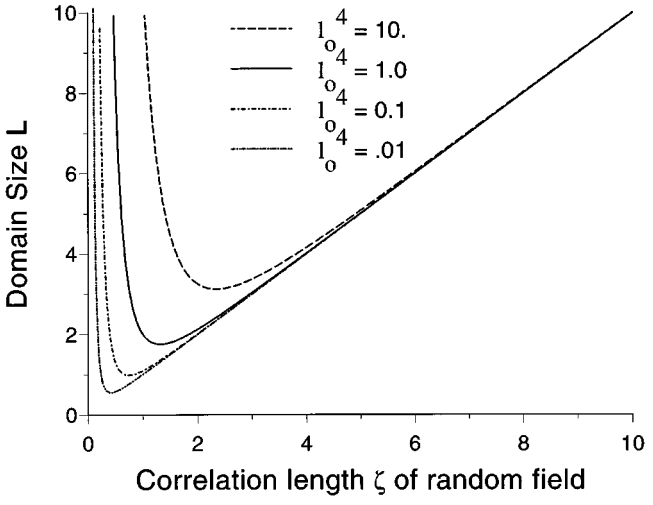


FIG. 1. Domain size vs correlation length of random field in arbitrary units. The coherence length $l_0$ increases with decreasing field strength.

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